Equations to remember

Equations not given in the formula sheet

- *Completing the square*:

$$x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$

- *By completing the square for a quadratic equation, you can find its turning point*:

$$y = a(x + p)^2 + q$$
$$turning\ point = (-p, q)$$

- *Gradient of a straight line graph*

$$m = \frac{Rise}{Run} = \frac{y_b - y_a}{x_b - x_a}$$

- *Equation of a straight line graph*

$$y - y_1 = m(x - x_1)$$

- *When two straight line graphs are parallel*:

$$m_1 = m_2$$

- *When two straight line graphs are perpendicular*:

$$m_1 \times m_2 = -1$$

- *Midpoint between two points*:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Equation of a Circle

$$(x-a)^2+(y-b)^2=r^2$$

$(a,b) = Centre\ of\ the\ circle$
$r = radius\ of\ the\ circle$

Midpoint of a diameter is the centre, can use the following:

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Distance between two points (x_1, y_1) and (x_2, y_2) can be found using pythagoras:

$$Distance = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Tangent & Chord properties

- A tangent is a straight line that touches the point of the outside of a circle(or curve)
  - The tangent to a circle is perpendicular to its radius at the point of intersection.
- A chord is a line from one point of the circumference to another, NOT passing the centre.
  - The perpendicular bisector of a chord will go through the centre

Logarithms

$$a^x = n \iff \log_a n = x$$

Laws of Logarithms

$$\log_a x + \log_a y = \log_a xy$$

$$\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$$

$$\log_a (x^k) = k \log_a x$$

$$\log_a \left(\frac{1}{x}\right) = \log_a (x^{-1}) = -\log_a (x)$$

$$\log_a a = 1$$

$$\log_a 1 = 0$$

Changing Log base

$$\log_a x = \frac{\log_b x}{\log_b a} \qquad a \longrightarrow b$$

$$\log_a b = \frac{1}{\log_b a}$$

Binomial Expansion - Factorial Trick

$$\binom{7}{1} = \frac{7}{1}$$

$$\binom{7}{2} = \frac{7 \times 6}{1 \times 2}$$

$$\binom{7}{3} = \frac{7 \times 6 \times 5}{1 \times 2 \times 3}$$

$$\binom{7}{4} = \frac{7 \times 6 \times 5 \times 4}{1 \times 2 \times 3 \times 4}$$

Trigonometric Identities

$$\tan x = \frac{\sin x}{\cos x}$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$$

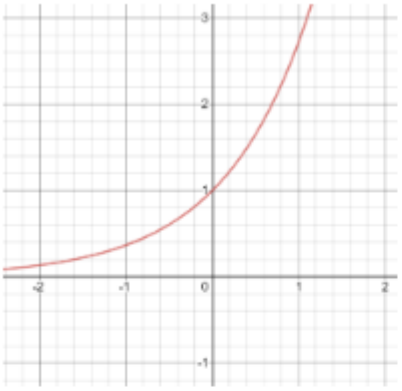
$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\cos^2 x = 1 - \sin^2 x$$

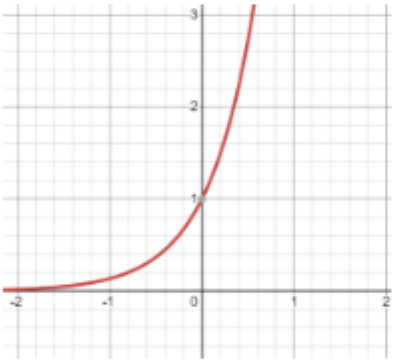
Transformations of $y = a^x$

$Y = a^x$

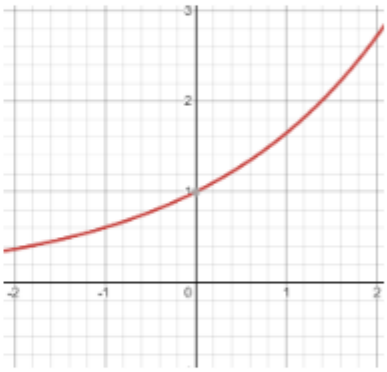


$Y = a^{-x}$

$Y = a^{2x}$



$Y = a^{1/2\,x}$

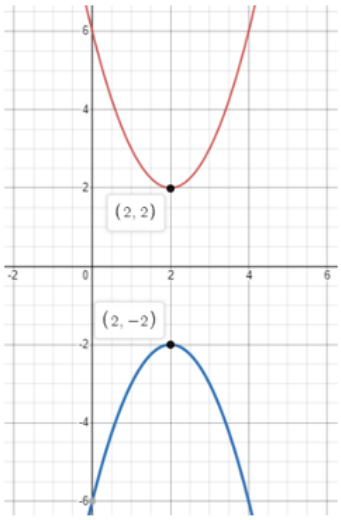


$Y = 2 + a^x$

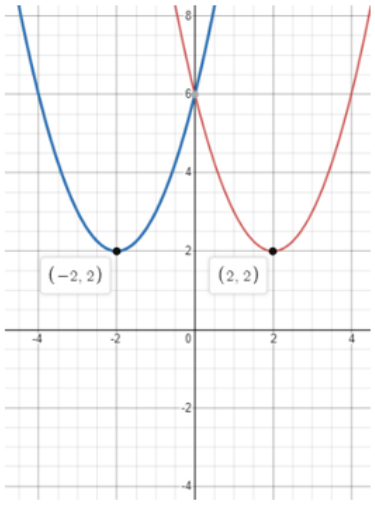
$Y = a^{x+2}$

Translating Graphs

$y = -f(x)$
is the reflection at the $x-$ *axis*



$y = f(-x)$
is the reflection at the $y-$ *axis*



Sketching Derivative Graphs

$y = f(x)$	$y = f'(x)$
Maximum or minimum	Cuts the x-axis
Point of inflection	Touches the x-axis
Positive gradient	Above the x-axis
Negative gradient	Below the x-axis
Vertical asymptote	Vertical asymptote
Horizontal asymptote	Horizontal asymptote at the x-axis

Proving the Sum of Arithmetic Series

Prove that the sum of the first n terms of an arithmetic series is $\frac{n}{2}(2a + (n-1)d)$.

$S_n = a + (a + d) + (a + 2d) + \ldots$
$\quad + (a + (n-2)d) + (a + (n-1)d) \quad (1)$

Write out the terms of the sum.

$S_n = (a + (n-1)d) + (a + (n-2)d) + \ldots.$
$\quad + (a + 2d) + (a + d) + a \quad (2)$

This is the sum reversed.

Adding (1) and (2):

Adding together the two sums.

$2 \times S_n = n(2a + (n-1)d)$

$S_n = \frac{n}{2}(2a + (n-1)d)$

Problem-solving

You need to learn this proof for your exam.